Kennedy Baptist College

Semester One Examination, 2023

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 3

Section Two: Calculator-assumed

SOLUTIONS

WA student number: In figures

In words

Your name

Number of additional answer booklets used (if applicable):

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	48	35
Section Two: Calculator-assumed	12	12	100	90	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (90 Marks)

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8 **(6 marks)**

A particle moves in space with position vector $\underset{\sim}{r}(t) = \begin{pmatrix} 2\cos(4t) \\ -3t \\ -2\sin(4t) \end{pmatrix}$ cm, where t is the time in seconds since its motion began.

(a) Determine the distance of the particle from its initial position after $\frac{\pi}{3}$ seconds. (3 marks)

Solution
$\underset{\sim}{r}\left(\frac{\pi}{3}\right) - \underset{\sim}{r}(0) = \begin{pmatrix} -1 \\ -\pi \\ \sqrt{3} \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ -\pi \\ \sqrt{3} \end{pmatrix}$ $\left\lvert \begin{pmatrix} -3 \\ -\pi \\ \sqrt{3} \end{pmatrix} \right\rvert = \sqrt{\pi^2 + 12} \approx 4.68 \text{ cm}$
Specific behaviours
✓ position vectors at $t = 0$ and required time   ✓ displacement vector   ✓ correct distance

(b) Show that the particle is moving with a constant speed. (3 marks)

Solution
Velocity vector: $\underset{\sim}{v}(t) = \frac{d}{dt}\left(\underset{\sim}{r}(t)\right) = \begin{pmatrix} -8\sin(4t) \\ -3 \\ -8\cos(4t) \end{pmatrix}$ Speed: $\lvert \underset{\sim}{v}(t) \rvert = \sqrt{(-8\sin(4t))^2 + (-3)^2 + (-8\cos(4t))^2}$ $= \sqrt{64\sin^2(4t) + 64\cos^2(4t) + 9}$ $= \sqrt{64 + 9}$ $= \sqrt{73} \approx 8.54 \text{ cm/s}$ Hence particle is moving with a constant speed.
Specific behaviours
✓ correct velocity vector   ✓ correct expression for magnitude of vector   ✓ simplifies magnitude to show constant

Question 9 **(6 marks)**

Particles A and B are moving with constant velocities and have initial positions $\begin{pmatrix} -8 \\ 2 \\ 10 \end{pmatrix}$ m and $\begin{pmatrix} 7 \\ 7 \\ -15 \end{pmatrix}$ m respectively. 2 seconds later A is at $\begin{pmatrix} 0 \\ -2 \\ 4 \end{pmatrix}$ m.

(a) Determine the velocity of A. (1 mark)

Solution
$\underset{\sim}{v}_A = \frac{1}{2}\left(\begin{pmatrix} 0 \\ -2 \\ 4 \end{pmatrix} - \begin{pmatrix} -8 \\ 2 \\ 10 \end{pmatrix}\right) = \begin{pmatrix} 4 \\ -2 \\ -3 \end{pmatrix}$
Specific behaviours
✓ correct velocity

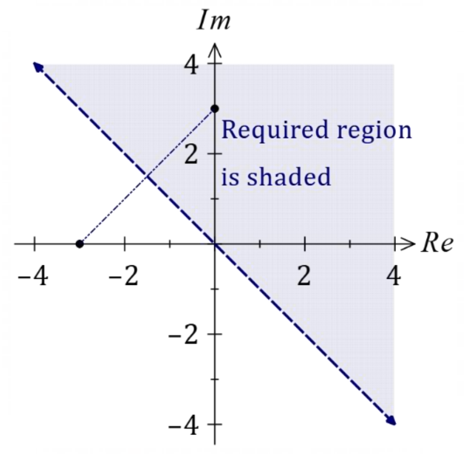
The velocity of B is $\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}$ m/s.

(b) Show that the paths of A and B cross, state the position vector of this point, and explain whether the particles collide. (5 marks)

Solution
$\underset{\sim}{r}_A(t) = \begin{pmatrix} -8 \\ 2 \\ 10 \end{pmatrix} + t\begin{pmatrix} 4 \\ -2 \\ -3 \end{pmatrix}, \quad \underset{\sim}{r}_B(s) = \begin{pmatrix} 7 \\ 7 \\ -15 \end{pmatrix} + s\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}$ For paths to cross we require $\underset{\sim}{r}_A = \underset{\sim}{r}_B$. Equating $\underset{\sim}{i}$ and $\underset{\sim}{j}$ coefficients and solving simultaneously: $4t - 8 = s + 7, \quad 2 - 2t = 7 - 3s \Rightarrow t = 5, s = 5$ Check $\underset{\sim}{k}$ coefficients are equal with these values of t and s: $t = 5 \Rightarrow 10 - 3(5) = -5, \quad s = 5 \Rightarrow -15 + 2(5) = -5$ Because $\underset{\sim}{r}_A(5) = \underset{\sim}{r}_B(5) = \begin{pmatrix} 12 \\ -8 \\ -5 \end{pmatrix}$, their paths cross at this point and because both particles reach this point at the same time they collide.
Specific behaviours
✓ indicates equations for both paths   ✓ forms two equations using different time parameters   ✓ solves equations and checks third coefficient   ✓ correct position vector   ✓ explains why paths cross and whether particles collide

Question 10 **(9 marks)**

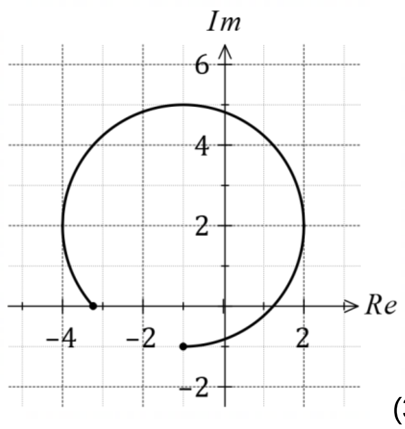
(a) Draw the subset of the complex plane determined by $|z+3| > |z-3i|$ on the axes below. (3 marks)



Solution
See diagram
Specific behaviours
✓ indicates points in plane
✓ draws perp' bisector with dotted line
✓ shades correct region

(b) The circular arc in the diagram represents the locus of a complex number z.

Without using $Re(z)$ or $Im(z)$, write equations or inequalities in terms of z for the indicated locus.



(3 marks)

Solution
Circle has centre $-1+2i$ and radius 3. $$\|z-(-1+2i)\| = 3 \cap \left(-\frac{3\pi}{4} \le \arg z \le \pi\right)$$
Specific behaviours
✓ indicates correct centre and radius
✓ writes inequality for circle
✓ writes restriction for $\arg z$

(c) Describe the subset of the complex plane determined by $|z-3| + |z+3i| = 3\sqrt{2}$. (3 marks)

Solution
Distance between 3 and $-3i$ in complex plane is $3\sqrt{2}$.   Hence z must lie on the line segment between 3 and $-3i$ inclusive in the complex plane.   Alternatively, when $z = x + iy$ then locus is $y = x - 3, 0 \le x \le 3$.
Specific behaviours
✓ indicates distance between points
✓ indicates subset is a line segment
✓ correct description that includes endpoints

Question 11 **(8 marks)**

(a) Determine the equations of all asymptotes of the graph of $y = f(x)$ when

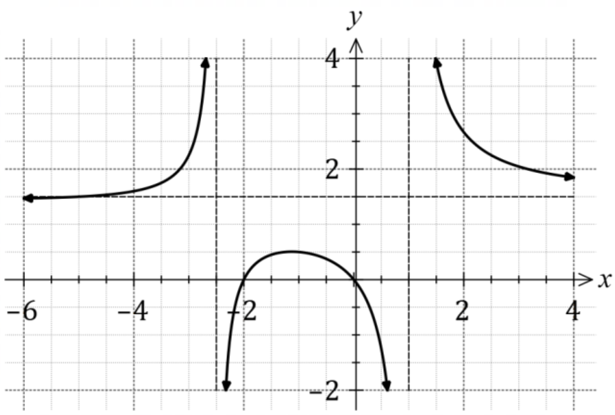
(i) $f(x) = \dfrac{1 + 2x^2}{x(1 - 3x)}$. (2 marks)

Solution
$f(x) = \dfrac{2x^2 + 1}{-3x^2 + x}$, $\lim\limits_{x \to \pm\infty} f(x) = -\dfrac{2}{3}$ Asymptotes: $x = 0,\ x = 1/3,\ y = -2/3$.
Specific behaviours
✓ horizontal asymptote   ✓ all asymptotes

(ii) $f(x) = \dfrac{x^2 + 4}{x - 5}$. (2 marks)

Solution
$f(x) = \dfrac{x^2 + 4}{x - 5} = x + 5 + \dfrac{29}{x - 5}$ Asymptotes: $x = 5,\ y = x + 5$.
Specific behaviours
✓ oblique asymptote   ✓ all asymptotes

(b) The graph of $y = g(x)$ is shown in the diagram, together with its three asymptotes.



The defining rule is given by

$$g(x) = \frac{ax(x + b)}{(2x + c)(x - d)}$$

where a, b, c and d are positive integer constants.

Determine, with brief reasons, the value of a, b, c and d. (4 marks)

Solution
Asymptote $y = 1.5 \rightarrow a/2 = 1.5 \rightarrow a = 3$.   Root at $(-2, 0) \rightarrow b = 2$.   Asymptote $x = -2.5 \rightarrow c = 5$.   Asymptote $x = 1 \rightarrow d = 1$.
Specific behaviours
✓✓✓✓ each value with appropriate reason

Question 12 **(7 marks)**

Relative to an origin O located on level ground, a projectile is launched from $\begin{pmatrix} 0 \\ 7.5 \end{pmatrix}$ m with an initial velocity of $\begin{pmatrix} 20 \\ 21 \end{pmatrix}$ m/s. The motion of the projectile is only affected by a constant acceleration of $\begin{pmatrix} 0 \\ -9.8 \end{pmatrix}$ m/s^2.

(a) Derive from the acceleration vector an expression for the position vector $\underset{\sim}{r}(t)$ of the projectile t s after its launch. (3 marks)

Solution
$\underset{\sim}{v}(t) = \int \underset{\sim}{a}(t)\, dt$ $= \begin{pmatrix} 0 \\ -9.8t \end{pmatrix} + \underset{\sim}{c}$ and $\underset{\sim}{v}(0) = \begin{pmatrix} 20 \\ 21 \end{pmatrix}$ $= \begin{pmatrix} 20 \\ 21 - 9.8t \end{pmatrix}$ $\underset{\sim}{r}(t) = \int \underset{\sim}{v}(t)\, dt$ $= \begin{pmatrix} 20t \\ 21t - 4.9t^2 \end{pmatrix} + \underset{\sim}{c}$ and $\underset{\sim}{r}(0) = \begin{pmatrix} 0 \\ 7.5 \end{pmatrix}$ $= \begin{pmatrix} 20t \\ 7.5 + 21t - 4.9t^2 \end{pmatrix}$
Specific behaviours
✓ antidifferentiates acceleration, shows constant   ✓ antidifferentiates velocity, shows constant   ✓ correctly uses initial conditions to evaluate constants

(b) Determine the distance travelled through the air by the projectile from when it is launched until the instant it reaches the ground, correct to the nearest 0.1 m. (4 marks)

Solution
Reaches ground level when vertical component of position is 0: $7.5 + 21t - 4.9t^2 = 0 \Rightarrow t = \dfrac{10\sqrt{3} + 15}{7} \approx 4.6172 \text{ s}$ Distance travelled: $d = \int_0^{4.6172} \lvert \underset{\sim}{v}(t) \rvert\, dt$ $= \int_0^{4.6172} \sqrt{20^2 + (21 - 9.8t)^2}\, dt$ $= 109.6 \text{ m}$
Specific behaviours
✓ equation for time to reach ground level   ✓ obtains time to reach ground level   ✓ integral for distance travelled   ✓ correct distance travelled

Question 13 **(7 marks)**

At time t seconds, $t \geq 0$, the position vector $\underset{\sim}{r}(t)$ m of a particle is given by

$$\underset{\sim}{r}(t) = \begin{pmatrix} 2e^{-0.5t} - 3 \\ 1 - 4e^{-1.5t} \end{pmatrix}$$

(a) State the position vector of the point that the particle approaches as $t \to \infty$. (1 mark)

Solution
$\lim\limits_{t\to\infty} \underset{\sim}{r}(t) = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$
Specific behaviours
✓ correct position vector

(b) Determine the speed of the particle when $t = 4$, correct to the nearest 0.001 m/s. (3 marks)

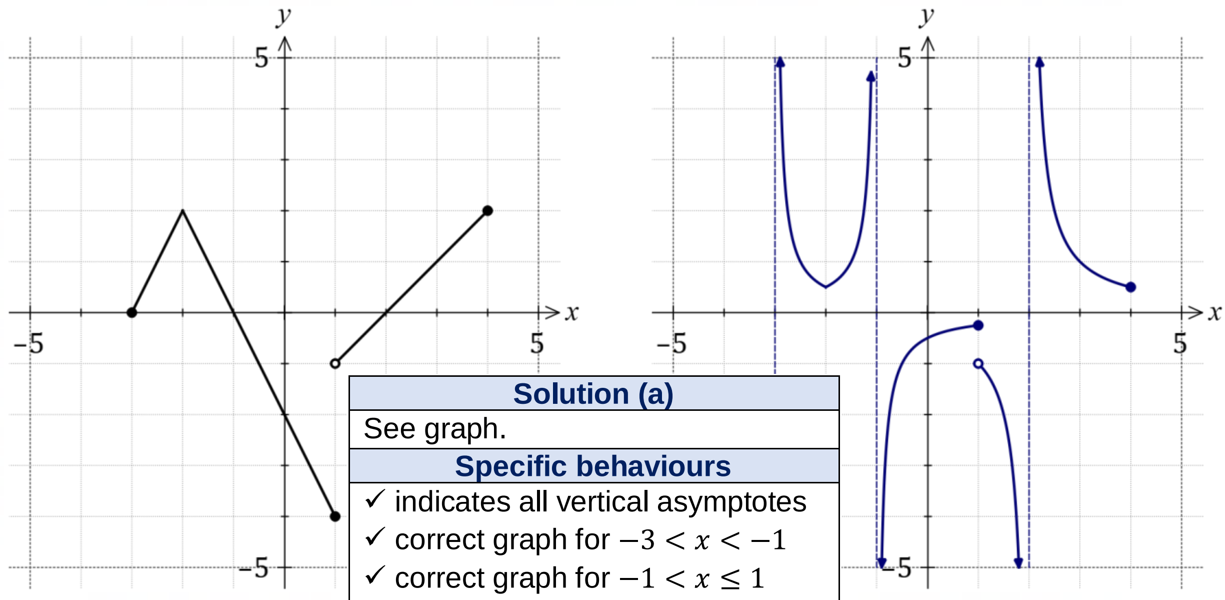
Solution
$\underset{\sim}{v}(t) = \begin{pmatrix} -e^{-0.5t} \\ 6e^{-1.5t} \end{pmatrix}$ $\underset{\sim}{v}(4) = \begin{pmatrix} -e^{-2} \\ 6e^{-6} \end{pmatrix}$ $\left\|\underset{\sim}{v}(4)\right\| = \sqrt{e^{-4} + 36e^{-12}}$ $= e^{-6}\sqrt{e^{8} + 36} \approx 0.136 \text{ m/s}$
Specific behaviours
✓ obtains velocity vector ✓ velocity vector at required time ✓ calculates magnitude of velocity

(c) Express the Cartesian equation for the path of the particle in the form $y = f(x)$. (3 marks)

Solution
$x = 2e^{-0.5t} - 3$ $e^{-0.5t} = \dfrac{x+3}{2}$ $\therefore e^{-1.5t} = \left(\dfrac{x+3}{2}\right)^3$ $y = 1 - 4e^{-1.5t}$ $= 1 - 4\left(\dfrac{x+3}{2}\right)^3$ $y = 1 - \dfrac{1}{2}(x+3)^3 \quad \text{where} \quad -3 < x \leq -1.$
Specific behaviours
✓ obtains expression for $e^{-0.5t}$ in terms of x ✓ obtains $y = f(x)$, simplification optional ✓ uses initial position and part (a) to state domain restriction

Question 14 **(9 marks)**

The graph of $y = f(x)$ is shown on the left-hand axes in the diagram below.



Solution (a)
See graph.
Specific behaviours
✓ indicates all vertical asymptotes
✓ correct graph for $-3 < x < -1$
✓ correct graph for $-1 < x \leq 1$
✓ correct graph for $1 < x < 2$
✓ correct graph for $2 < x \leq 4$

(a) Sketch the graph of $y = \dfrac{1}{f(x)}$ on the right-hand axes in the diagram. (5 marks)

(b) Solve the following equations.

(i) $f(|x|) = 1$. (1 mark)

Solution
For $x \geq 0, f(x) = 1 \Rightarrow x = 3,\ \therefore x = \pm 3.$
Specific behaviours
✓ correct solution set

(ii) $\left|\dfrac{1}{f(x)}\right| = 1.$ (1 mark)

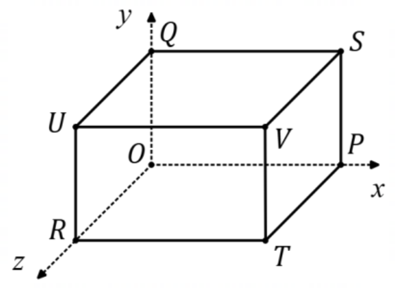
Solution
$\left\|\dfrac{1}{f(x)}\right\| = 1 \Rightarrow f(x) = \pm 1 \Rightarrow x = -2.5, -1.5, -0.5, 3$
Specific behaviours
✓ correct solution set

(iii) $|f(x)| + f(x) = 0.$ (2 marks)

Solution
Roots and intervals where $f(x) \leq 0$: $(x = -3) \cup (-1 \leq x \leq 2).$
Specific behaviours
✓ includes 3 roots
✓ correct solution set

Question 15 **(6 marks)**

The diagram shows a right rectangular prism.



Relative to vertex O, vertices P, Q and R have position vectors $\begin{pmatrix} p \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ q \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ r \end{pmatrix}$.

(a) Determine vectors $\overrightarrow{PU}$ and $\overrightarrow{QT}$ in terms of p, q and r. (1 mark)

Solution
$\overrightarrow{PU} = \begin{pmatrix} -p \\ q \\ r \end{pmatrix}, \quad \overrightarrow{QT} = \begin{pmatrix} p \\ -q \\ r \end{pmatrix}$
Specific behaviours
✓ correct vectors

(b) Use a vector method to show that diagonals PU and QT bisect each other. (3 marks)

Solution
Midpoint of line PU: $$\underset{\sim}{m_1} = \overrightarrow{OP} + \frac{1}{2}\overrightarrow{PU} = \begin{pmatrix} p \\ 0 \\ 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -p \\ q \\ r \end{pmatrix} = \frac{1}{2}\begin{pmatrix} p \\ q \\ r \end{pmatrix}$$ Midpoint of line QT: $$\underset{\sim}{m_2} = \overrightarrow{OQ} + \frac{1}{2}\overrightarrow{QT} = \begin{pmatrix} 0 \\ q \\ 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} p \\ -q \\ r \end{pmatrix} = \frac{1}{2}\begin{pmatrix} p \\ q \\ r \end{pmatrix}$$ Since $\underset{\sim}{m_1} = \underset{\sim}{m_2}$ then diagonals are coincident at their midpoints and so bisect each other.
Specific behaviours
✓ develops expression for position vector of one midpoint ✓ develops expression for position vector of one midpoint ✓ shows midpoints are coincident and hence bisect

(c) Determine the relationship between p, q and r when $\overrightarrow{PU}$ and $\overrightarrow{QT}$ are perpendicular. (2 marks)

Solution
For vectors to be perpendicular, require $\begin{pmatrix} -p \\ q \\ r \end{pmatrix} \cdot \begin{pmatrix} p \\ -q \\ r \end{pmatrix} = 0$. Hence $-p^2 - q^2 + r^2 = 0$ or $r^2 = p^2 + q^2$.
Specific behaviours
✓ indicates condition for perpendicularity ✓ correct relationship

Question 16 **(8 marks)**

(a) Determine all solutions to the equation $z^3 - 8i = 0$ in exact polar form. (3 marks)

Solution
$z^3 = 8\,\text{cis}\left(\frac{\pi}{2}\right) \Rightarrow z = 2\,\text{cis}\left(\frac{\pi + 4n\pi}{6}\right), n = -1, 0, 1$ $z_1 = 2\,\text{cis}\left(-\frac{\pi}{2}\right), \quad z_2 = 2\,\text{cis}\left(\frac{\pi}{6}\right), \quad z_3 = 2\,\text{cis}\left(\frac{5\pi}{6}\right)$
Specific behaviours
✓ expresses $8i$ in polar form   ✓ states one correct solution   ✓ states all correct solutions

(b) Consider the ninth roots of unity expressed in polar form $r\,\text{cis}\,\theta$.

(i) Determine the roots for which $0 < \theta < \frac{\pi}{2}$. (2 marks)

Solution
$z^9 = 1 = \text{cis}(2n\pi) \Rightarrow z = \text{cis}\left(\frac{2n\pi}{9}\right)$ where $n \in Z$.   Hence $z_1 = \text{cis}\left(\frac{2\pi}{9}\right), \quad z_2 = \text{cis}\left(\frac{4\pi}{9}\right).$
Specific behaviours
✓ general expression for roots   ✓ correct roots

(ii) Use all nine roots to show that $\cos\left(\frac{2\pi}{9}\right) + \cos\left(\frac{4\pi}{9}\right) + \cos\left(\frac{8\pi}{9}\right) = 0.$ (3 marks)

Solution
The nine roots are given by $z = \text{cis}\left(\frac{2n\pi}{9}\right), n = -4, -3, \ldots, 3, 4$, and the sum of these roots, and hence their real parts, will be 0: $\cos(0) + \cos\left(\frac{2\pi}{9}\right) + \cos\left(\frac{4\pi}{9}\right) + \cos\left(\frac{6\pi}{9}\right) + \cos\left(\frac{8\pi}{9}\right) + \cos\left(-\frac{2\pi}{9}\right) + \cos\left(-\frac{4\pi}{9}\right) + \cos\left(-\frac{6\pi}{9}\right) + \cos\left(-\frac{8\pi}{9}\right) = 0$ But $\cos(-\theta) = \cos(\theta), \cos\left(\frac{6\pi}{9}\right) = -\frac{1}{2}$ and $\cos(0) = 1$. Hence $1 + 2\cos\left(\frac{2\pi}{9}\right) + 2\cos\left(\frac{4\pi}{9}\right) - 1 + 2\cos\left(\frac{8\pi}{9}\right) = 0$ $\therefore \cos\left(\frac{2\pi}{9}\right) + \cos\left(\frac{4\pi}{9}\right) + \cos\left(\frac{8\pi}{9}\right) = 0$
Specific behaviours
✓ uses sum of real parts of all roots is 0 ✓ uses $\cos(-\theta) = \cos(\theta)$ and known values   ✓ simplifies to obtain required result

Question 17 **(7 marks)**

Plane Π has equation $2x - y - 3z = 13$ and point A has coordinates $(1, 5, 4)$.

(a) Determine the coordinates of the point in Π that is closest to A. (4 marks)

Solution
Equation of line through $A \perp \Pi$ is $\underset{\sim}{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix}$. Intersects with plane $2x - y - 3z = 13$ when $$2(1 + 2\mu) - (5 - \mu) - 3(4 - 3\mu) = 13 \Rightarrow \mu = 2$$ $$\underset{\sim}{r} = \begin{pmatrix} 1 \\ 5 \\ 4 \end{pmatrix} + 2 \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix}$$ Coordinates are $(5, 3, -2)$.
Specific behaviours
✓ equation of line through point ✓ uses intersection to obtain equation in μ ✓ solves for μ ✓ correct coordinates

Vector $\underset{\sim}{v}$ lies in plane Π, is perpendicular to the line $\dfrac{x+1}{-1} = \dfrac{y-3}{2} = \dfrac{z-1}{2}$ and $\left|\underset{\sim}{v}\right| = \sqrt{26}$.

(b) Let $\underset{\sim}{v} = a\underset{\sim}{i} + b\underset{\sim}{j} + c\underset{\sim}{k}$. Determine the value of coefficients a, b and c, given that $a > c$. (3 marks)

Solution
$$\underset{\sim}{v} \text{ lies in plane} \Rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} = 0 \Rightarrow 2\mathrm{a} - \mathrm{b} - 3\mathrm{c} = 0$$ $$\underset{\sim}{v} \perp \text{line} \Rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} = 0 \Rightarrow -\mathrm{a} + 2\mathrm{b} + 2\mathrm{c} = 0$$ $$\left|\underset{\sim}{v}\right| = \sqrt{26} \Rightarrow a^2 + b^2 + c^2 = 26$$ Solving equations simultaneously: $\underset{\sim}{v} = \pm \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$ $$a = 4, \quad b = -1, \quad c = 3.$$
Specific behaviours
✓ two equations using normals ✓ equation using magnitude ✓ correct set of values

Question 18 **(9 marks)**

Let $u = \sqrt{3} + i$ and $v = \sqrt{2}\,\text{cis}\left(\frac{\pi}{30}\right)$.

(a) Determine an exact value for

(i) $\arg(uv)$. (1 mark)

Solution
$\arg(uv) = \arg u + \arg v = \frac{\pi}{6} + \frac{\pi}{30} = \frac{\pi}{5}$
Specific behaviours
✓ correct value

(ii) $\lvert u + i \rvert$. (1 mark)

Solution
$\lvert u + i \rvert = \lvert \sqrt{3} + 2i \rvert = \sqrt{3 + 4} = \sqrt{7}$
Specific behaviours
✓ correct value

(b) Let $w = \frac{u^4}{v^n}$, where n is a positive integer.
Determine the minimum value of n so that w is purely imaginary. (3 marks)

Solution
For $Re(w) = 0$ then $\arg w = \pm\frac{\pi}{2}$. $\arg w = 4\arg u - n\arg v = \frac{4\pi}{6} - \frac{n\pi}{30} = \frac{(20-n)\pi}{30}$ $\frac{(20-n)\pi}{30} = \frac{\pi}{2} \Rightarrow n = 5$
Specific behaviours
✓ expression for $\arg w$ ✓ indicates values of $\arg w$ for $Re(w) = 0$ ✓ correct value of n

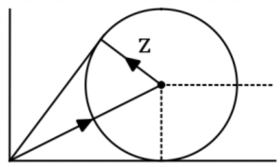
The modulus of complex number z is 1 and its argument is θ, where $-\pi < \theta \leq \pi$.

(c) Determine the value of θ for which

(i) $|u + z|$ is minimum. (1 mark)

Solution
For $|u + z|$ to be minimum, u and z must be parallel but in opposite direction. Hence $$\theta = \frac{\pi}{6} - \pi = -\frac{5\pi}{6}$$
Specific behaviours
✓ correct value

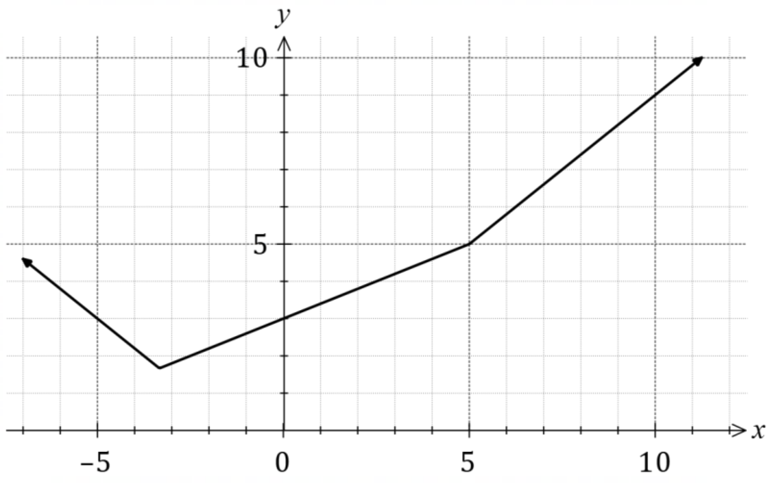
(ii) $\arg(u + z)$ is maximum, where $-\pi < \arg(u + z) \leq \pi$. (3 marks)

Solution	
Locus of $u + z$ is circle, centre u and radius 1. Maximum $\arg(u + z) = \frac{\pi}{3}$, and from geometric considerations this occurs when $\theta = \frac{5\pi}{6}$.	
Specific behaviours	
✓ sketch diagram (possibly seen in part(b)(i)) ✓ indicates z for maximum argument ✓ correct value	

Question 19 **(8 marks)**

Let $f(x) = |ax + b| + |cx + d|$ where a, b, c and d are constants such that $a \geq c \geq 0$.

The graph of $y = f(x)$ is shown below and passes through the points $(0,3), (5,5)$ and $(10,9)$.



(a) The equation $f(x) = kx + 1$ has an infinite number of solutions. State the value of the constant k. (1 mark)

Solution
k is slope of RH part of f. $k = \frac{4}{5} = 0.8$.
Specific behaviours
✓ correct value

(b) Determine the value of a, b, c and d. (5 marks)

Solution

Equation of RH part of f is $y = 0.8x + 1$ and since $a \geq c \geq 0$ then

$$(a + c)x + (b + d) = 0.8x + 1 \Rightarrow a + c = 0.8, \qquad b + d = 1$$

Equation of central part of f is $y = 0.4x + 3$ and so either

$$(a - c)x + (b - d) = 0.4x + 3 \text{ or } (c - a)x + (d - b) = 0.4x + 3.$$

If $c - a = 0.4$ then $c = a + 0.4$ but this is impossible given that $a \geq c$.

Solving $a + c = 0.8$ and $a - c = 0.4$ gives $a = 0.6, c = 0.2$.

Solving $b + d = 1$ and $b - d = 3$ gives $b = 2$ and $d = -1$.

Values: $a = 0.6,\ b = 2,\ c = 0.2,\ d = -1$.

Specific behaviours

✓ uses RH part to form equations for $a + c$ and $b + d$
✓ uses central part to form equations for $a - c$ and $b - d$
✓ repeats for $c - a$ and $d - b$ and eliminates impossible pair of equations
✓ correct values for a and c
✓ correct values for b and d

(c) Determine the minimum value of $f(x)$. (2 marks)

Solution
$-0.8x - 1 = 0.4x + 3 \Rightarrow x = -\dfrac{10}{3} \Rightarrow f\left(-\dfrac{10}{3}\right) = \dfrac{5}{3}$
Specific behaviours
✓ indicates correct method to obtain x-coordinate ✓ correct minimum

End of questions

Supplementary page

Question number: _________

Supplementary page

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